



## COURSE DESCRIPTION CARD - SYLLABUS

Course name

Mathematical analysis III [S1MNT1>AM3]

### Course

Field of study

Mathematics of Modern Technologies

Year/Semester

3/6

Area of study (specialization)

–

Profile of study

general academic

Level of study

first-cycle

Course offered in

Polish

Form of study

full-time

Requirements

compulsory

### Number of hours

Lecture

30

Laboratory classes

0

Other

0

Tutorials

30

Projects/seminars

0

### Number of credit points

4,00

### Coordinators

prof. dr hab. inż. Paweł Kolwicz  
pawel.kolwicz@put.poznan.pl

### Lecturers

### Prerequisites

Knowledge of mathematical analysis from previous semesters. The ability to efficiently determine integrals, calculate derivatives, analyze the function of a real variable. He is aware of the need to expand his competences and is ready to cooperate.

### Course objective

Getting to know the concept of function variation and the Riemann-Stieltjes integral, getting to know and using the Lebesgue measure, the measure in the  $\sigma$ -algebra of sets, the ability to act on measurable functions, getting to know the general notion of the integral and using it for the Lebesgue integral, learning about the relationships between the Riemann integral and the Lebesgue integral, the ability to analyze various types of convergence of function sequences.

### Course-related learning outcomes

Knowledge:

- has the knowledge of the concept of function variation and the Riemann-Stieltjes integral, the concept of Lebesgue measure, the concept of algebra and  $\sigma$ -algebra of sets, the concept of a measure in the  $\sigma$ -algebra of sets, the definition of a measurable function and the integral with respect to the measure

[K\_W01(P6S\_WG)];

- understands the differences between various types of convergence of function sequences (pointwise convergence, almost everywhere convergence, uniform convergence) [K\_W01(P6S\_WG)];
- is aware of the relationships between the measure and integral theory with the theory of Banach function spaces [K\_W01(P6S\_WG)].

Skills:

- is able to determine the variation of the function and the Riemann-Stieltjes integral [K\_U01(P6S\_UW)];
- can determine the Lebesgue measure of sets, determine the measures of sets in the  $\sigma$ -algebra, determine the integral with respect to the measure, the Lebesgue integrals (simple examples) [K\_U01(P6S\_UW)];
- can verify various types of convergence of function sequences [K\_U01(P6S\_UW)].

Social competences:

- can think and act mathematically correct in the area of measure and integral theory [K\_K01(P6S\_KK)];
- knows the limitations of his own knowledge and understands the need for further education, understands the need for systematic work [K\_K02(P6S\_KK)].

### Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

Lectures:

passing threshold: 50% of the points; the issues for the exam, on the basis of which the questions are developed, will be sent to students by e-mail using the university's electronic systems;

- assessment of knowledge and skills on a written exam checking the knowledge of notions and the ability to prove theorems and illustrate theories with examples (also possible short practical tasks);
- obtaining additional points for activity during lectures, including for the presentation of papers (discussing additional aspects of the issues, in particular the application of the discussed theory in other sciences or a reference to the location in the history of mathematics) and for comments on improving teaching materials;

Tutorials:

the knowledge acquired during the exercises is verified by two tests carried out around the 7th and 15th week (alternatively 1 test at the end of the semester); passing threshold: 50% of the points; the rules for completing the course and the exact grading thresholds will be provided to students at the beginning of the semester with the use of university electronic systems);

- continuous assessment - rewarding activity (additional points) manifested in the discussion and cooperation in solving practical tasks;
- continuous assessment - rewarding the increase in the ability to use the techniques learned;
- obtaining additional points for activity during classes, including for the presentation of papers (discussing additional aspects of the issues, in particular the application of the discussed theory in other sciences or a reference to the location in the history of mathematics) and for comments on improving teaching materials;
- active participation in consultations deepening knowledge and directing further work.

### Programme content

Update: 01.06.2024r.

Lectures:

theoretical issues (definitions, lemmas, theorems, conclusions, algorithms, properties) and relevant examples for the following issues:

- finite variation functions and the Riemann Stieltjes integral
- the Lebesgue measure
- the measure space
- measurable functions
- integral with respect to measure

the relationships of the theorems with the theory of Banach function spaces which is currently being developed,

Tutorials:

solving practical problems illustrating the notions discussed and studying examples with the use of

theoretical machinery from the lecture, e.g.: determining the variation and the Riemann-Stieltjes integral, checking the measurability of sets, determining measures, analyzing classic examples of measures, checking the convergence of function sequences, determining the integral with respect to the measure, using the known theorems in the context of the examples considered.

## Course topics

### Lecture

#### I. Functions of finite variation and the Riemann-Stieltjes integral

functions of bounded variation (relations between finite variation and important properties - boundedness, continuity, Lipschitz condition)

Riemann-Stieltjes integral (properties and more important theorems, applications to curvilinear integrals and in probabilistics);

#### II. Lebesgue measure

Open set measure, outer Lebesgue measure, measurable set in the sense of Lebesgue, properties, examples

#### III. The measure space

algebra and  $\sigma$ -algebra of sets, measurable space, measure, Borel measure, complete measure, continuity of measure

#### IV. Measurable functions

Equivalence conditions for measurability of functions, characteristic function and simple function, probabilistic measure space, pointwise convergence, convergence almost everywhere, by measure and uniform convergence, relations between types of convergence, Yegorov's theorem, Luzin's theorem, Frechet's theorem

#### V. The integral with respect to the measure

The integral of a simple function, the integral of a function with respect to measure, the relationship between integral in the Riemann sense and integral in the Lebesgue sense, Lebesgue's Theorem of Majorized Convergence, Fatou's Lemma, the relationship of these theorems to the contemporaneous theory of functional Banach spaces, measure and integral in the Cartesian product and Fubini's Theorem

### Tutorials

Solving practical tasks using the material presented in the lecture

## Teaching methods

### Lectures:

- a lecture conducted on the blackboard in an interactive way with the formulation of questions to a group of students;
- the activity of students is taken into account (preparation of historical papers on mathematicians related to the presented material, presentations on the relationship between the subject and the theory of Banach function spaces, presentation of proofs left to be done on their own) during classes when giving the final grade;
- initiating discussions during the lecture;
- theory presented in connection with the current knowledge of students from previous lectures.

### Tutorials:

- solving example tasks on the blackboard;
- detailed reviewing of the solutions to the tasks by the tutor and discussion of the comments.

## Bibliography

### Basic:

- H. J. Musielak, *Analiza matematyczna, tom II, część 1*, Wydawnictwo Naukowe UAM, Poznań 1999;
- J. Musielak i M. Jaroszewska, *Analiza matematyczna, tom II, część 2*, Wydawnictwo Naukowe UAM,

Poznań 2002;

• J. Musielak i M. Jaroszevska, Analiza matematyczna, tom II, część 3, Wydawnictwo Naukowe UAM, Poznań 2002.

Additional:

• S. Hartman i J. Mikusiński, Teoria miary i całki Lebesguea, Państwowe Wydawnictwo Naukowe, Warszawa 1957;

• W. Rudin, Podstawy analizy matematycznej, Państwowe Wydawnictwo Naukowe, Warszawa 2000;

• W. Rudin, Analiza rzeczywista i zespolona, Państwowe Wydawnictwo Naukowe, Warszawa 1986.

### Breakdown of average student's workload

	Hours	ECTS
Total workload	100	4,00
Classes requiring direct contact with the teacher	62	2,50
Student's own work (literature studies, preparation for laboratory classes/ tutorials, preparation for tests/exam, project preparation)	38	1,50